Neutrino Lensing

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Abstract. Due to the intrinsic properties of neutrinos, the gravitational lens effect for neutrino should be more colorful and meaningful than the normal lens effect of photon. Other than the oscillation experiments operated at terrestrial laboratory, in principle, we can propose a completely new astrophysical method to determine not only the nature of gravity and spacetime of lens objects but also the mixing parameters of neutrinos by analyzing neutrino trajectories near the central objects. However, compared with the contemporaneous telescopes through the observation of the electromagnetic radiation, the angular, energy and time resolution of the neutrino telescopes are still comparatively poor, we just concentrate on the two classical tests of general relativity, i.e. the angular deflection and time delay of neutrino by a lens object as a preparative work in this paper. In addition, some simple properties of neutrino lensing are investigated.

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Neutrinos are one kind of the fundamental particles that make up the universe. Up to date, many interesting neutrino astrophysical sources have been known, such as the ultra-low energy Big Bang relic background neutrinos, the ~ 10 keV stellar neutrinos, the ~ 10 MeV supernova neutrinos, the ~ 100 TeV neutrinos from young supernova remnants in our galaxy, the ultra-high energy neutrinos from Active Galactic Nucles (AGNs), Gamma Ray Bursts (GRBs), Microquasars or some other celestial sources. Since neutrinos are electrically neutral leptons, they only participate in the weak interaction and are almost unaffected through transmission, and they open a new window into space to the high-energy processes in the universe to astronomy. As we known, several under-ice and under-water neutrino telescopes are being or scheduled to be constructed now, such as Icecube located at the south Pole worked in energy range from GeV to PeV, expected to achieve an angular resolution below $1°$, a real-time resolution at $2\,\mathrm{ns}$ and an energy resolution below 0.3 in $\log(E)$.

In addition, neutrinos are also hot topics and research areas for particle physicists. One of the most remarkable discoveries in particle physics connect with neutrinos in the last few decades is the finding that neutrinos are massive and mixed particles. Although the absolute scale of neutrino masses is still unknown, the evidence is strong. The precision measurement of neutrino masses and oscillation parameters now is one of

the main goals of the neutrino experiments. It is possible for researchers to measure the absolute mass of neutrino with an accuracy of 0.05 eV soon by the double beta decay experiment in many laboratories or by the lensing of the Cosmic Microwave Background radiation (CMB) from cosmological probes.

As we known, a light ray can be deflected by gravity, it is same for neutrino. Since Soldner firstly published the derivation of the deflection of light by a massive object based on the framework of Newtonian gravity almost two centuries ago,[1] the gravitational lensing has been full investigated numerically or analytically. Now, lensing of light ray has been used widely in the research of astrophysics and already became an excellently educational stuff for students of astronomy.[2] Another kind of gravitational lens researchers less mentioned is neutrino lensing, which has also been put forward and widely studied for a quite long time. Unlike photon, neutrino can penetrate into most of celestial objects easily except black holes or some special compact objects since it only participates in the gravitational and weak interaction. Therefore, one obvious advantage of neutrino lensing in principle is that it may provide a distinct method to probe the mass distribution of lens objects. Based on the standard solar model, Gerver investigated the possibility of neutrino focusing by the Sun's core.[3] Escribano *et al.*[4] discussed the gravitational lensing of neutrinos by some extended sources as stars, galaxies and galactic halos. Moreover, Mena *et al.*[5] studied the lensing effect of supernova neutrinos by the center black hole of our Galaxy. Eiroa and Romero[6] analyzed the lensing effect of the cosmological sources of neutrinos by some supermassive black holes. However, all of works cited here used the lightray approximation, treated neutrino as a massless particle. Strictly speaking, neutrino does not move along null geodesics since it has a non-zero rest mass. Based on this idea, Barrow and Subramanian[7] offered a possible explanation for the time delay of neutrinos from supernova SN1987a.

In this Letter, we will show that this small but non-zero rest masses of neutrino will bring us some tinily small but non-trivial difference of the angular deflection and time delay of neutrino in gravitational field. In principle, neutrino lensing itself may provide a unique ground for probing the nature of neutrinos. Thus, we investigate gravitational lensing of neutrinos by the point-mass lens and re-investigate two famous effects of General Relativity, i.e. the deflection and the Shapiro time delay, we extend them to more general case.

We start from the Kerr metric in Boyer-Lindquist (BL) coordinates

$$ds^2 = \rho^2(\frac{dr^2}{\Delta}+d\theta^2)+(r^2+a^2)\sin^2\theta d\phi^2-c^2dt^2+\frac{r_s r}{\rho^2}(a\sin^2\theta d\phi-cdt)^2 \ , \tag{1}$$

where $r_s = 2GM/c^2$ is the Schwarzschild radius, $a = J/Mc$ is the specific angular momentum with the dimension of length, $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - r_s r + a^2$. Since this metric describe a stationary and axisymmetric spacetime, it is easy for us to obtain those conserved quantities for test particles in Kerr geometry,

$$E/mc = (1-\frac{r_s r}{\rho^2})c\dot{t} + \frac{r_s r}{\rho^2}a\sin^2\theta\dot{\phi} \ , \tag{2}$$

$$L_z/m = -\frac{r_s r}{\rho^2} a \sin^2\theta c\dot{t} + \frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\rho^2} \sin^2\theta\dot{\phi} \, . \tag{3}$$

The dots denote differentiation with respect to an affine parameter. The constants E/m and L_z/m can be viewed as the energy and the axial component of angular momentum per unit mass at infinity. In addition, other two constants of motion are the particle's rest mass m,

$$mc = |p| = \left(-g^{\mu\nu} p_\mu p_\nu\right)^{1/2} \, , \tag{4}$$

and Carter's constant Q,

$$Q = p_\theta^2 + \cos^2\theta \left[a^2(m^2c^2 - E^2/c^2) + (L/\sin\theta)^2\right] \, . \tag{5}$$

For simplicity, we only investigate the motion for the open (or infinite) orbits in the equatorial plane, i.e. $E \geq mc^2$, $\theta = \pi/2$ and $p_\theta = 0$. Considering an incoming particles with Lorentz factor $\gamma = E/mc^2$and impact parameter $b \approx L_z/(\sqrt{\gamma^2-1}mc)$ at distance, we can solve for the velocity components in terms of γ and b

$$\dot{t} = \frac{1}{\Delta}\left[\left(r^2 + a^2 + \frac{a^2 r_s}{r}\right)\gamma - \frac{a r_s}{r} b\sqrt{\gamma^2-1}\right] \, , \tag{6}$$

$$\dot{\phi} = \frac{c}{\Delta}\left[\frac{a r_s}{r}\gamma + \left(1 - \frac{r_s}{r}\right) b\sqrt{\gamma^2-1}\right] \, , \tag{7}$$

$$\dot{r}^2 = \frac{c^2}{r^3}\left[\gamma^2 r^3 + r_s\left(a\gamma - b\sqrt{\gamma^2-1}\right)^2 + r\left(a^2\gamma^2 - b^2\gamma^2 + b^2\right)\right] - \frac{c^2\Delta}{r^2} \, . \tag{8}$$

It is obvious that the above equations are not only suitable for null geodesics for massless particles like the photon ($\gamma \to \infty$) but also suitable for time-like geodesics for massive particles with finite γ. The orbit of a photon is more simple since it is not decided by its energy but completely decided by its impact parameter. We will focus on the general behavior of test particles (including photons) hereafter.

Setting $dr/d\phi = 0$, we can derive a relationship between the turning point (the point of closest approach to central object) r_0 and the impact parameter b,

$$b_\pm = \left[\sqrt{\gamma^2-1}(1 - \frac{r_s}{r_0})\right]^{-1}\left[-a\gamma\frac{r_s}{r_0} \pm r_0\sqrt{(1 - \frac{r_s}{r_0} + \frac{a^2}{r_0^2})(\gamma^2 - 1 + \frac{r_s}{r_0})}\right] \, , \tag{9}$$

where $+$ and $-$ denote prograde and retrograde particles respectively. We also can express r_0 as a power series in $u = r_s/|b|$,

$$\begin{aligned} r_0/|b| = 1 &- \frac{1+s^2}{2s^2}u - \left(\frac{3}{8} + \frac{1}{4s^2} - \frac{1}{8s^4} - \frac{\tilde{a}\sqrt{1+s^2}}{s} + \frac{\tilde{a}^2}{2}\right)u^2 \\ &- \left[\frac{1}{2} + \frac{1}{2s^2} - \frac{\tilde{a}\sqrt{1+s^2}\,(1+3s^2)}{2s^3} + \frac{1}{2}\tilde{a}^2\left(2 + \frac{1}{s^2}\right)\right]u^3 \\ &- \left[\frac{105s^8 + 140s^6 + 30s^4 - 4s^2 + 1}{128s^8} - \frac{\tilde{a}\sqrt{1+s^2}\,(2+3s^2)}{s^3}\right. \\ &\left. + \frac{\tilde{a}^2\,(51s^4 + 46s^2 + 3)}{16s^4} - \frac{\tilde{a}^3\sqrt{1+s^2}}{s} + \frac{\tilde{a}^4}{8}\right]u^4 + \mathcal{O}(u^5) \, , \end{aligned} \tag{10}$$

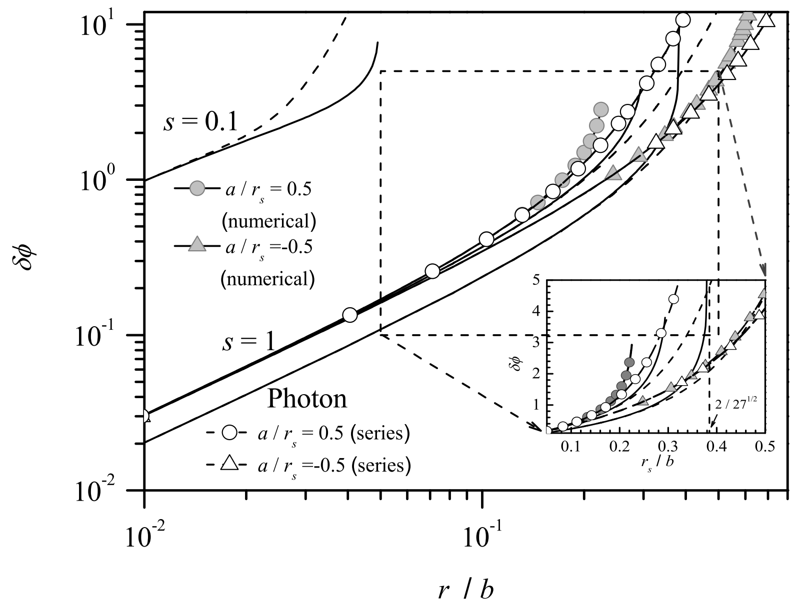

Figure 1. Deflection angle for photon or some other relativistic particles with different impact parameters. The solid and the dashed curves denote the numerical solutions and the series solutions, respectively. The circle and the triangle curves represent the deflection of the retrograde and the prograde particles (with $s = 1$ as a demonstration) in the equatorial plane of the extreme Kerr Black Hole.

where $s = \sqrt{\gamma^2 - 1}$, $\tilde{a} = a/r_s$ is the specific angular momentum in units of the Schwarzschild radius, with $\tilde{a} > 0$ for prograde particles and $\tilde{a} < 0$ for retrograde particles in a unified expression.

Simply and repeatedly used the mathematical relation $\partial_u \left[\int_0^{f(u)} g(x,u)\, dx\right] = \int_0^{f(u)} \partial_u[g(x,u)]\, dx + f'(u)\, g(f(u),u)$, the deflection angle (in radians) can be expressed as a Taylor series about u as

$$\begin{aligned}
\delta\phi &= 2\int_{r_0}^{\infty} \left(\dot{\phi}/\dot{r}\right) dr - \pi \\
&= \left(2 + \frac{1}{s^2}\right) u + \left[\frac{3}{16}\,\pi(5 + \frac{4}{s^2}) - \frac{2\tilde{a}\sqrt{1+s^2}}{s}\right] u^2 + \frac{1}{6}\left[\left(32 + \frac{36}{s^2} + \frac{6}{s^4} - \frac{1}{2s^6}\right)\right. \\
&\quad \left. - \frac{\pi\tilde{a}\sqrt{1+s^2}\,(2+5s^2)}{2s^3} + \left(2 + \frac{1}{s^2}\right)\tilde{a}^2\right] u^3 + \left[\frac{105\,\pi}{1024}\left(33 + \frac{48}{s^2} + \frac{16}{s^4}\right)\right. \\
&\quad \left. - \frac{3\tilde{a}\sqrt{1+s^2}\,(1 + 12s^2 + 16s^4)}{2s^5} + \frac{3\pi\tilde{a}^2(8 + 88s^2 + 95s^4)}{64s^4} - \frac{2\,\tilde{a}^3\sqrt{1+s^2}}{s}\right] u^4 \\
&\quad + \mathcal{O}(u^5)\,. \qquad (11)
\end{aligned}$$

If you do not care about the interminable derivation and integration, definitely you can expand the above integration to any order you want. As a demonstration, geodesics are expanded as a Taylor series up to and including fourth-order terms in $r_s/|b|$ here. When $a = 0$ and $s \to \infty$, we recover the famous first order light deflection in Schwarzschild space-time, $2r_s/b$, and add several high order modification. Fig.1 shows the logarithm

plot of the deflection angle for photon and some other relativistic particles with different impact parameters. For the convenience of comparison, we have carried out the deflection integral numerically, i.e. the solid curves in Fig.1. Clearly, the Taylor series expansion offered here are quite well formula to specify the bending angle especially for small angle, weak field (with $|b|$ much larger than r_s) and ultra-relativistic particles (with $s \gg 1$). The intermediate relativistic particles with $s = 1$ are taken as an example to demonstrate the rotational contribution. It is natural and obvious that the spin is a high-order modification, which only appears in second-order or even higher order terms of the expansion, which means that the spin parameter of black hole only can be determined by the careful observation of the large angle scattering of the intermediate or ultra relativistic particles. Due to the frame-dragging effect of the rotating black holes, the particle traveling in the direction of rotation of the object will move around the central object faster than particles moving against the rotation, the position of the turning point for prograde particle will be even more far away from the center than that of retrograde particles. Thereupon the deflection angle is enhanced for retrograde particles (the circle curves) and reduced for prograde particles (the triangle curves). The vertical dashed line in the infixed plot, $r_s/b = 2/\sqrt{27}$, corresponding to the critical impact parameters b_c for Schwarzschild black hole. For $b \leq b_c$, the photon will spiral in and be captured by the central black hole. For more general case in Schwarzschild spacetime, the critical value is

$$b_c = r_s \, \frac{\sqrt{\left(3\gamma^2 - 2\right)^2 \left(3\gamma^2 - 4\right) + \gamma\sqrt{9\gamma^2 - 8}\left(9\gamma^4 - 20\gamma^2 + 12\right)}}{\sqrt{2}\left(\gamma^2 - 1\right)\left(\sqrt{9\gamma^2 - 8} - \gamma\right)} \, . \tag{12}$$

However there is no analytic expression for the critical impact parameter in Kerr black hole. By considering the capture of the background photons, hot, warm and cold dark matter in the early Universe, we plan to investigate the growth of primordial black holes in our future work, which are possible seeds for supermassive black holes at the center of the galaxies.

We have discussed the angular deflection of photon or relativistic particle by a rotating spherical body. From the first order term of Eq. (11), we can see that the difference of the deflection between photon and ultra-relativistic particle, $2r_s/b(\gamma^2 - 1)$, is tinily small, which means that the neutrino lens and the photon lens are nearly the same in weak deflection limit, that is why people can safely and simply use null geodesics as photons do to study neutrino lensing.[5, 6] If we want to directly determine the mass of neutrinos by bending, we must seek for the observation of the large angle scattering. Unlike photons, neutrinos are electrical neutrality, only participate in the weak interaction, can penetrate the lens objects easily, hence it should be possible for researchers to detect those large angle scattering neutrinos in principle. However, it is difficult for us to find the optical counterpart of the same source to compare with due to the strong dust extinction in the neighborhood of the lens objects. Furthermore, the requirement for angular resolution may go far beyond the capability of neutrino telescopes at present even in the near future, the operation process to detect the bending

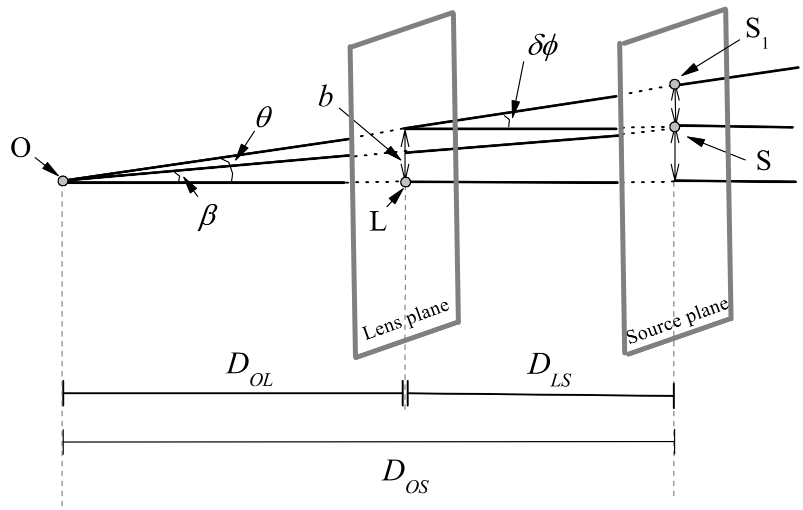

Figure 2. Sketch of a typical lensing system.

angle precisely will be very difficult and nearly hopeless for us. Thus, it seems that our work here is more meaningful from the theoretical than from the observational point of view. In fact, we really plan to study the neutrino lensing by a Kerr black hole in weak deflection limit with higher order modification by following Sereno's work[8] in a forthcoming paper. In the following paragraphs, we only devote to the study of the properties of neutrino lensing as the simplest, i.e. the point-mass lens model is employed. In addition, we only consider the contribution of the first order term for simplicity.

If there are no other deflectors except a lensing object of mass M close to the line-of-sight to a source, the sketch map of a typical lensing system can be drawn easily (See Fig. 2). The distances between source and observer, source and lens, and observer and lens are given by D_{OS}, D_{LS}, D_{OL}, respectively. The angle β and θ denote the positions of source and images with respect to the lens direction. If the physical size of the lensing object is much smaller than both D_{LS} and D_{OL}, and the all angles (β, θ, $\delta\phi$) are very small, from Fig. 2 we can reach the geometric relation $\beta = \theta - \delta\phi\, D_{LS}/D_{OS}$. Notice $b = \theta\, D_{OL}$ and the deflection angle expression to first order, we obtain the famous lens equation $\beta = \theta - \theta_E^2/\theta$, where the Einstein angle θ_E now satisfies $\theta_E^2 = (2 + 1/s^2)\, r_s D_{LS}/(D_{OS} D_{OL})$, which defines the angular scale for a lens situation. By solving the lens equation, one can find the positions of the primary (upper sign) and the secondary (lower sign) images for an isolated point source, $\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$, which correspond to the shortest geodesic and the longest geodesic with $b_1 = \theta_1 D_{OL}$ and $b_2 = |\theta_2| D_{OL}$, respectively.

As we known, times can be measured to much greater accuracy than angles, we will further investigate another prominent and equivalent effect of General Relativity, i.e. the Shapiro time delay or gravitational time delay here. This effect was first pointed by Shapiro in 1964 about the time delay of signals passing near a massive object.[9] Let us

calculate the time required for a particle to go form a source point with $r = r_S$, $\phi = \phi_S$, to an observer at $r = r_O$, $\phi = \phi_O$ in the equatorial plane. The equation governing the time history of orbits is easy to be given by Eq. (6) and (8). The time required for particle to go from the turning point r_0 to r is

$$\begin{aligned} t(r, r_0) &= \int_{r_0}^{r} \left(\dot{t}/\dot{r}\right) dr \\ &\simeq \frac{\sqrt{r^2 - r_0^2}}{c\,\sqrt{\gamma^2 - 1}\big/\gamma} + \frac{r_s}{2c}\frac{\gamma^3}{\left(\gamma^2 - 1\right)^{3/2}} \left\{ \sqrt{\frac{r - r_0}{r + r_0}} + \frac{2\gamma^2 - 3}{\gamma^2} \cdot \ln\left(\frac{r + \sqrt{r^2 - r_0^2}}{r_0}\right) \right. \\ &\quad - \frac{a\left(a - 2a\gamma^2 + 2\gamma\sqrt{\gamma^2 - 1}\sqrt{a^2 + r_0^2}\right)}{\gamma^2 r_0^2}\sqrt{\frac{r - r_0}{r + r_0}} - \frac{\left(\gamma^2 - 1\right)^{3/2}}{\gamma^3} \cdot \\ &\quad \left. \ln\left[\frac{\left(a^2 + r^2\right) r_0^2}{r^2 r_0^2 + a^2\left(2r^2 - r_0^2\right) - 2ar\sqrt{\left(r^2 - r_0^2\right)\left(a^2 + r_0^2\right)}}\right] \right\} . \end{aligned} \tag{13}$$

We only keep the first order in r_s/r in the integrand function. The result show that the traveled the time has two different components. The leading term is related to the geometrical of the spacetime in the absence of the lens, now is the time for the relativistic particle traveled in straight lines at unit velocity, $c\sqrt{\gamma^2 - 1}/\gamma$, in the Minkowski spacetime. The other terms are the first-order approximation for the gravitational time delay. This is the well-known 'Shapiro effect', which has been amply tested by radar echo delay experiments in our Solar System. Of course, the total time required for a particle to go form a source point to an observer can be written as $t_{OS} = t(D_{OL}, r_0) + t(\sqrt{D_{LS}^2 + D_{OS}^2\beta^2}, r_0)$.

Since particles that form distinct images are almost emitted at the same time from the same source but travel by different paths, they may reach an observer at different time. Combining Eq. (13) with the above image equation, we can infer the physical time delay function between the gravitationally lensed images,

$$\begin{aligned} \Delta t_{im} &= t_{OS2} - t_{OS1} \\ &\simeq \frac{r_s}{c}\frac{\gamma\left(2\gamma^2 - 1\right)}{2\left(\gamma^2 - 1\right)^{3/2}} \left[2\bar{\beta}\sqrt{1 + \frac{1}{4}\bar{\beta}^2} + \frac{2\gamma^2 - 3}{2\gamma^2 - 1}\ln\left(\frac{1 + \frac{1}{2}\bar{\beta}^2 + \bar{\beta}\sqrt{1 + \frac{1}{4}\bar{\beta}^2}}{1 + \frac{1}{2}\bar{\beta}^2 - \bar{\beta}\sqrt{1 + \frac{1}{4}\bar{\beta}^2}}\right)\right] , \end{aligned} \tag{14}$$

where $\bar{\beta} = \beta/\theta_E$ is the reduced misalignment angle. Furthermore, we can give the time delay of a relativistic particle relative to a photon emitted by the same source at the same time, Δt_{ph}, which can be measured very easy. As space is limited, we shall not go into details of the formula.

Besides the multiplicity and distortion of images and the time delay between different images, another prominent effect of lensing is the magnifications of the images, which are given by the ratio of the solid angles of the image and the source,

$$\mu_{\pm} = \left|\frac{\theta_{\pm} d\theta_{\pm}}{\beta d\beta}\right| = \frac{\bar{\beta}^2 + 2}{2\bar{\beta}\sqrt{\bar{\beta}^2 + 4}} \pm \frac{1}{2} . \tag{15}$$

The measurable total magnification $\mu = \mu_+ + \mu_-$ is always larger than one. Since the Einstein angle θ_E now is a slightly larger than that of photon, the image magnification of neutrino lensing is slightly larger than that of normal lensing. As usual, the maximal magnification is infinite as $\beta \to 0$, but this is not a real situation since the physical objects should have a finite size $R_\star$, the effective limit now is $\beta \to \beta_\star = R_\star/D_{OS}$.

Based on 17-year radial velocity and 12-year astrometric measurements of the short period star S0-2, last year Ghez *et al.*[10] reported the newest and the most precise results about the central supermassive black hole in our galaxy. The balk hole is 8.0 ± 0.6 kpc from us with mass $(4.1 \pm 0.6) \times 10^6 M_\odot$. Supposed a type of two supernova explode at 8 kpc away from our galactical center with anggular position $\beta = 0.1 \sim 0.5\,\theta_E$. The order of magnitude of the time delay between the primary and secondary images (no matter for photons or neutrinos) is several dozens seconds. Recently experimental results (from KamLAND, MINOS, Super-Kamiokande, etc.) and CMB observation show neutrinos have sub-eV mass (at least one mass eigenstate state neutrino with a mass of at least $0.04\,\mathrm{eV}$, the best estimate of the mass square difference are $\Delta m_{21}^2 = 7.684 \times 10^{-5}\,\mathrm{eV}^2$, $\Delta m_{32}^2 = 0.0027\,\mathrm{eV}^2$).[11],[12],[13] Taken $E_\nu \sim 10\,\mathrm{Mev}$, the typical value of supernova neutrinos, $m_\nu c^2 \sim 1\,\mathrm{eV}$, the flight time delay of neutrino relative to photon is about $\sim 10^{-5}\mathrm{s}\,\frac{\mathrm{D_{OS}}}{16\mathrm{kpc}}\sqrt{1+\sin^2\beta}$. Thus, we can really use the forthcoming neutrino facilities to do some more detailed analysis of time-varying. Combined with the observation of the normal lensing of light ray, neutrino lensing probably can provide us some valuable hints about the spacetime characteristic of the lens object, even the intrinsic quality of neutrino itself. However we only investigate the point-mass lens model, while the matter distribution during the propagation may has a strong impact on the results, so a more detailed and sophisticated treatments are need in the future.

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